Abstract

We develop an integrated approach for analyzing logistics and marketing decisions within the context of designing an optimal returns system for a retailer servicing two distinct market segments. At the operational level, we show that the optimal refund price is not unique. Moreover, it is such that if both market segments return a purchased product, then neither segment will receive a full money-back refund; and it is such that if one or both segments do not return a purchased product, then a refund premium over the purchase price is possible, but the refund premium will not be enough to offset a customer’s total net cost of purchase and return. We also show that any improvement to the returns system that results in increased logistical efficiency or marketing effectiveness will be accompanied by an increase in the selling price of the product. At the strategic level, we show that if the retailer does not coordinate its logistics and marketing efforts to improve the overall returns system, then it will tend to over-invest in one of the functions and under-invest in the other. Finally, we illustrate how our model can be generalized to the case in which a customer’s ex post valuation of the product falls along a continuum.

Keywords: Retailing; Product returns; Operations/marketing interface; Money-back guarantees

1. Introduction

With the trend towards improving customer satisfaction and the emergence of retailers on the Internet, product returns systems have become an important part of business. In a survey of 40 e-commerce marketing executives reported by Forrester Research, 30% considered online returns to be one of their biggest challenges (McCullough Kilgore et al., 1999). Some e-retailers have even teamed with companies like UPS to provide services that reduce handling costs while allowing customers to return products more efficiently.
and with less hassle. For example, buy.com reports a partnership with UPS that has significantly reduced the time between a customer’s request for a return and the actual return, thereby helping the customer return a product much faster and at a much smaller cost (UPS Press Release, September 20, 2000). One reason for the increasing interest in product returns is that customers cannot always fully observe what they are buying on the Internet and this results in a higher possibility that a given customer will purchase a product that does not match her specific needs. Knowing that this is the case, companies are becoming more sensitive towards designing more profitable product returns systems.

We identify three components of an integrated product returns system that can improve a company’s bottom line. The first component is the refund policy. In any purchase, even if the customer is well informed about the product, there is a possibility that after taking the product home the customer might realize that the product is not exactly what was expected. The customer obviously has some needs and is seeking fulfillment of those needs by buying the product. However, if the product turns out not to be what the customer thought it to be prior to the purchase, then those needs are not fully satisfied and the value of the product is reduced in the eyes of this customer. The customer, aware of this risk from the beginning, will be reluctant to go ahead with the purchase unless there is some protection mechanism. A refund policy provides such protection by allowing a customer to spend some time with the product before making a final decision. As a result, a refund policy decreases a customer’s risk associated with making a purchase, and thereby increases the total demand for the product.

The second component is the logistics process. When a return occurs, both the retailer and the customer experience costs. For the customer, there is the transaction cost associated with returning a product (e.g., the expense and hassle of shipping) and for the retailer there is the handling cost associated with processing the return (e.g., the cost of repackaging). Thus, an efficient logistics process could result in either or both of two effects: Like a refund policy, it could increase total demand by reducing the customer’s cost associated with making a purchase; or it could increase the average profit margin by reducing direct costs.

The third component is the marketing initiative to sharpen market segmentation. Recall that, in any purchase, there is some probability that the product will not match what a given customer thought it to be prior to the purchase. However, the greater the a priori information content of the product’s characteristics, the better segmented the market will become, and thus, the higher the probability that a given purchase will result in a match between the product’s properties and the customer’s needs. Thus, an effective marketing initiative could result in an increased average number of matches per sale.

In this paper, we develop and analyze a model for integrating these three components of a product returns system (Fig. 1). Our objective is two-fold: To determine a decision vector representing a jointly optimal refund policy, logistics process, and marketing initiative and to develop corresponding insights. Our model consists of a single retailer making sales in a single period. Any given sale results either in a match or a mismatch, but the market is imperfectly segmented so neither the retailer nor a given customer knows with certainty which will result from a particular sale (otherwise, returns would not occur). The retailer’s decisions are to establish the refund price, the amount to invest to reduce the total per unit shipping and handling cost (retailer plus customer) associated with a return, and the amount to invest to better segment the market (and thereby increase the probability that a sale will result in a match). These three decisions represent the retailer’s refund policy, logistics process, and marketing initiative, respectively. We also discuss the impact on our model and its analysis if alternative decision proxies are substituted for those defined here.

The primary contribution of our paper is that it provides an integrated framework for designing a product returns system that combines logistical efficiency with marketing effectiveness at the strategic level, and that coordinates a responsive refund policy at the operational level. Another contribution of our paper is that it succinctly synthesizes and extends previous research on refund policies. In particular, our simplified model of the customer returns process provides results complementary to those of similar models, yet it produces novel insights.
The remainder of this paper is organized as follows: In Section 2, we provide a review of related literatures. In Section 3, for the case in which there exists two distinct ex post customer valuations, we develop our base model and establish the retailer's optimal refund policy for any given logistics process and market segmentation. We also demonstrate how this model is linked to similar models appearing in the literature. Then, in Section 4, we extend our model to establish the retailer's corresponding optimal logistics and marketing strategies. In Section 5, we generalize our model and results for the case in which a customer's ex post valuation of the product falls along a continuum; and in Section 6, we investigate the cost of inefficient decision-making by comparing the jointly optimal logistics and marketing decisions to two sub-optimal measuring sticks motivated by the literature on organizational power and politics. Finally, we discuss the scope and applicability of our model in Section 7.

2. Literature review

The notion of “product returns” is found in three different yet linked streams of literature. The first stream deals with money-back guarantees offered by retailers to their customers. Focusing on the business-to-consumer (B2C) environment, these papers deal with the risk that the product may not match the pre-purchase expectations of a certain customer. The goal is to set a returns policy that maximizes the profits of the retailer. Particularly instrumental to the development of our model are the works by Davis et al. (1995) and by Davis et al. (1998). Davis et al. (1995) find that, in a market with homogeneous customers, a full money-back guarantee is desirable if the retailer has some amount of advantage over the customer in terms of salvaging the product, if the customer’s transaction cost of returning the product or the fraction of the total value of the product consumed during trial is low, or if the probability of a match between the product properties and the expectations of the customer is small. Then, Davis et al. (1998) extended these results to a situation in which the customer market is not homogeneous and the retailer can control the “hassle” involved in returning a product. Our paper builds off of these earlier works by allowing for a partial money-back guarantee, by incorporating the customer's transaction cost as a decision variable that can be manipulated by the retailer through a logistics investment function, and by exploring the implications of making the probability of match an endogenous variable. Accordingly, our results complement, extend, and generalize those of Davis et al. (1995, 1998).
Hess et al. (1996) introduce a non-refundable charge to dissuade moral hazard, which refers to the situation in which customers buy a product with the intention of returning it, just to get free value out of the product during the process. The authors find that the retailer is more inclined towards setting a non-refundable charge if the probability of a match, the trial value, or the overall valuation of the customer is high; or the transaction cost of the customer or the salvage value of a returned product is low. We too make use of a non-refundable charge but with a simpler model that extends these results. We also discuss refund policy issues that have not been discussed in the literature and introduce coordinated decisions in marketing and logistics.

The second stream of literature considers returns policies offered by manufacturers to retailers for goods that are unsold at the end of a demand period. These business-to-business (B2B) models differ from the B2C models in one crucial way: The B2C policies deal with a single customer and the uncertainty in product valuation or probability of a match, which is later translated into demand, while the B2B papers mainly deal directly with the uncertainty in the demand. Generally, the policies evaluated are in the form of full or partial money-back for some or all of unsold goods remaining at the retailer. Pasternack (1985) considers a manufacturer who tries to achieve channel coordination (maximization of total channel profits) when the retailer is self-serving and there is uncertainty in demand. He derives the following results: (1) For channel coordination, it is not optimal to allow the retailer to return everything at full credit or to allow no returns at all. (2) It is optimal to allow full returns at partial credit. (3) The total profits of the channel may be divided differently between the retailer and the manufacturer according to the type of policy set by the manufacturer. Result (2) is consistent with the results of both Hess et al. (1996) and this paper, which indicate the existence of situations in which a partial money-back guarantee is optimal.

Marvel and Peck (1995), Padmanabhan and Png (1997), and Emmons and Gilbert (1998) extend Pasternack’s work to consider channel coordination as an objective and to solve various versions of the returns problem, mostly concentrating on demand uncertainty and competing retailers. They investigate the tradeoffs between the risk of retailer overstocking if the policy is too generous and the threat of low sales if the policy is too strict.

The third stream of literature involving product returns is the warranty literature. Although this literature deals with different issues than the ones described above, it still has models that resemble the models that we have seen. Representative examples include Cooper and Ross (1985) and Lutz (1989). In this literature, there is again a possibility that a sold product may be returned; but the difference is that instead of including some uncertainty in customer valuation, demand, or the probability of match, these models deal with uncertainty in product quality and customer maintenance effort.

3. Base model: Optimal refund policy

In this section, we develop, solve, and analyze a model for determining the retailer’s optimal refund policy for a given logistics process and market segmentation. In Section 4, we extend the model to establish the retailer’s correspondingly optimal logistics and marketing strategies.

3.1. Consumers

We consider a single-period model in which a price-setting retailer sells a single product to a market consisting of two types of customers: (i) matched and (ii) mismatched. Matched customers are defined as customers who, after making a purchase, find the product to be consistent with their pre-purchase expectations. Matched customers have valuation $v_1$. Mismatched customers are defined as customers who, after making a purchase, find the product not to be consistent with their pre-purchase expectations. Mismatched customers have valuation $v_2$ ($0 \leq v_2 < v_1$). The fraction of customers who are matched is $\theta$, but
the market is imperfectly segmented in the sense that neither the retailer nor a given customer knows a priori whether a given sale will result in a match or a mismatch. Hence, \( \theta \) also can be interpreted as the probability that any given sale will result in a match. We are assuming here that the match between the product and any given customer's pre-purchase expectations is a Bernoulli variable to remain consistent with the literature. In Section 5, we relax this assumption.

3.2. Retailer

The retailer sells the product for price \( p \), but after a sale takes place, the customer is given a trial period to ascertain whether there is a match or a mismatch between the product and the customer's needs. The proportion of value of the product consumed by a given customer during the trial period is denoted by \( \alpha \) (\( 0 \leq \alpha \leq 1 \)), which also could be interpreted as the length of the trial period divided by the total useful lifetime of the product. We assume that \( \alpha \) is given and is fixed. This assumption would apply, for example, if the retailer and the manufacturer agree on a salvage value with the condition that the product has a proportion \((1 - \alpha)\) of its value remaining when it is returned to the manufacturer. We discuss implications of making \( \alpha \) a decision variable in Section 7.

The customer has the option of keeping or returning the product at the end of the trial period regardless of whether there is a match or mismatch. If the customer returns the product, then the retailer reimburses the customer with the refund price \( R \); in turn, the retailer receives a salvage value \( S \) (e.g., by selling the returned product back to the manufacturer). The total logistics cost of the exchange is \( T = h + c \), where \( h \) denotes the retailer's share of any shipping and handling expense incurred in the return and salvage of the product, and \( c \) denotes the customer's share.

3.3. Consumer purchase decision

If a customer makes a purchase, it could result either in a match or in a mismatch, and the customer could decide either to keep or to return the product at the end of the trial period. Thus, there are four possible scenarios that could result from a purchase. These scenarios are summarized by Fig. 2.

As illustrated in Fig. 2, the expected utility of a customer who makes a purchase has two components, one related to the case of a match and the other to the case of a mismatch. However, regardless of whether the purchase results in a match or a mismatch, type (i) customers can keep the product and derive a utility of \( v_i - p \), or return the product and derive a utility of \( \alpha v_i + R - c - p \), where \( \alpha v_i \) represents the value received during the trial period and \( R - c \) represents the net refund obtained by returning the product. Thus,

![Fig. 2. Post-purchase scenarios.](image)
the customer’s pre-purchase expected utility, which should be non-negative for the customer to make a purchase, is given by

\[ U(p, R) = \theta \max \{xv_1 + R - c, v_1\} + (1 - \theta) \max \{xv_2 + R - c, v_2\} - p. \] (1)

Notice that if \( R \) is pre-defined to equal \( p \) and if there is no risk of moral hazard (defined in Section 2), then (1) reduces to the Davis et al. (1995) model. If \( v_2 = 0 \), \( v_1 \) is a random variable, and \( R \) is pre-defined to equal \( p \), then (1) reduces to the Davis et al. (1998) model. Finally, if \( v_1 \) is uniformly distributed between 0 and \( V \), and \( R \) is pre-defined to equal \( p + c \), then (1) reduces to the Hess et al. (1996) model.

3.4. Retailer’s optimal decision

Let

\[ W = \text{the retailer’s purchase cost of the product (we make the natural assumption that } W > S, \text{ where, recall, } S \text{ is the retailer’s salvage value);} \]
\[ n = \text{the expected number of returns given that consumers maximize utility when deciding whether or not to return a purchased product (we normalize the number of sales to equal one so that } n \leq 1; \]
\[ II(p, R) = \text{the retailer’s expected profit of operations resulting from its pricing and refund policy, given a logistics process (defined by } c \text{ and } h \text{) and market segmentation (characterized by } \theta). \]

The retailer’s decision is to choose the values of \( p \) and \( R \) that maximize

\[ II(p, R) = p - W - (R + h - S)n \] (2)

subject to

\[ U(p, R) = \theta \max \{xv_1 + R - c, v_1\} + (1 - \theta) \max \{xv_2 + R - c, v_2\} - p \geq 0. \] (3)

From (2) and (3) we establish the following lemma, which we state without formal proof:

**Lemma 1.** For given values of the refund price \( R \), the retailer’s optimal selling price, \( p^* \), and the corresponding expected number of returns, \( n^* \), are determined as follows:

<table>
<thead>
<tr>
<th>If...</th>
<th>Then ( n^* = \ldots )</th>
<th>and ( p^* = \ldots )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R - c \leq (1 - x)v_2 )</td>
<td>0</td>
<td>( \theta v_1 + (1 - \theta)v_2 )</td>
</tr>
<tr>
<td>( (1 - x)v_2 &lt; R - c \leq (1 - x)v_1 )</td>
<td>( 1 - \theta )</td>
<td>( \theta v_1 + (1 - \theta)(xv_2 + R - c) )</td>
</tr>
<tr>
<td>( (1 - x)v_1 &lt; R - c )</td>
<td>1</td>
<td>( x[\theta v_1 + (1 - \theta)v_2] + R - c )</td>
</tr>
</tbody>
</table>

Intuitively, Lemma 1 follows because it is in the retailer’s best interest to set the price such that \( U(p, R) = 0 \) (since \( II(p, R) \) is increasing in \( p \)), and thereby extract the total expected utility from the market. Expected utility, however, depends on whether or not it is in the best interest of a given customer to return a purchased product. In turn, this depends on the comparison between \( R - c \), the net refund obtained by a customer if a purchased product is returned, and \( (1 - x)v_i \), the residual value received by a type \( i \) customer if the product is kept beyond the trial period. If \( R - c \leq (1 - x)v_2 \), then neither mismatched nor matched customers should return purchased products (i.e., \( n^* = 0 \)). We define this as the case of no returns (NR). If \( (1 - x)v_2 < R - c \leq (1 - x)v_1 \), then only mismatched customers should return purchased products (i.e., \( n^* = 1 - \theta \)); we define this as the case of some returns (SR). And, if \( (1 - x)v_2 < (1 - x)v_1 < R - c \), then both mismatched and matched customers should return purchased products (i.e., \( n^* = 1 \)). We refer to this as the case of all returns (AR).
Note that, ex post, a matched customer will receive a positive utility while a mismatched customer will receive a negative utility. This is intuitive since the expected utility of any customer is zero and a matched customer is guaranteed to receive more value than a mismatched customer. Substituting $p^*$ for $p$ in (2) reduces the retailer’s decision to a single-variable optimization problem in $R$. Accordingly, $R^*$, the retailer’s optimal refund price, is the value of $R$ that maximizes $\Pi(R) = \Pi(p^*, R)$, where, from Lemma 1 and (2), $\Pi(R)$ can be written as

$$\Pi(R) = \begin{cases} 
\theta v_1 + (1 - \theta)v_2 - W & \text{if } R \leq (1 - \alpha)v_2 + c, \\
\theta v_1 + (1 - \theta)(xv_2 - S) - W & \text{if } (1 - \alpha)v_2 + c < R \leq (1 - \alpha)v_1 + c, \\
x(\theta v_1 + (1 - \theta)v_2) - S - W & \text{if } (1 - \alpha)v_1 + c < R,
\end{cases}$$

(4)

and, recall, $T = c + h$. Notice that $\Pi(R)$ is not continuous in $R$. However, it is constant for each of the return scenarios identified in (4): NR, SR, or AR. Consequently, let $\Pi_j$ denote the constant value of $\Pi(R)$ associated with return scenario $j$ (for $j = \text{NR, SR, AR}$). Then, return scenario $j$ is optimal only if it satisfies two conditions: (i) $\Pi_j \geq \Pi_k$ for $k \neq j$ and (ii) $\Pi_j > 0$. If condition (ii) is not satisfied for any $j$, then no return scenario is optimal (in which case the optimal course of action for the retailer is not to offer the product for sale in the first place). Thus, a direct comparison between $\Pi_{\text{NR}}$, $\Pi_{\text{SR}}$, and $\Pi_{\text{AR}}$ indicates the optimal return scenario for the retailer. Theorem 1 provides the results. The corresponding values of $R^*$, $n^*$, $p^*$, and $\Pi' = \Pi(R^*)$ associated with each optimal return scenario then are summarized by Table 1. (Unless otherwise stated, the proofs to all theorems are in Appendix A.)

**Theorem 1.** For given values of $c$, $h$, and $\theta$, the retailer’s optimal return scenario is determined as follows (where $T = c + h$):

| If... | Then optimal return scenario is...
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$T &lt; S - (1 - \alpha)v_1$</td>
<td>AR</td>
</tr>
<tr>
<td>$S - (1 - \alpha)v_1 \leq T &lt; S - (1 - \alpha)v_2$</td>
<td>SR</td>
</tr>
<tr>
<td>$S - (1 - \alpha)v_2 \leq T$</td>
<td>NR</td>
</tr>
</tbody>
</table>

### 3.5. Insight and sensitivity

According to Theorem 1, the retailer’s optimal refund policy depends solely on what is in the best interest of a social planner whose objective is to maximize the total expected channel welfare (retailer profit plus market utility) associated with a return by a type (i) customer. To demonstrate, consider that the total channel cost associated with a return by a type (i) customer is $T$. And, the total channel expected benefit

<table>
<thead>
<tr>
<th>Return scenario</th>
<th>AR</th>
<th>SR</th>
<th>NR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimality condition</td>
<td>$T &lt; S - (1 - \alpha)v_1$</td>
<td>$S - (1 - \alpha)v_1 \leq T &lt; S - (1 - \alpha)v_2$</td>
<td>$S - (1 - \alpha)v_2 \leq T$</td>
</tr>
<tr>
<td>$R^*$</td>
<td>any $R$ s.t. $R &gt; (1 - \alpha)v_1 + c$</td>
<td>any $R$ s.t. $(1 - \alpha)v_2 + c &lt; R \leq (1 - \alpha)v_1 + c$</td>
<td>any $R$ s.t. $R \leq (1 - \alpha)v_2 + c$</td>
</tr>
<tr>
<td>$n^*$</td>
<td>1</td>
<td>$1 - \theta$</td>
<td>0</td>
</tr>
<tr>
<td>$p^*$</td>
<td>$\alpha(\theta v_1 + (1 - \theta)v_2) + R - c$</td>
<td>$\theta v_1 + (1 - \theta)(xv_2 + R - c)$</td>
<td>$\theta v_1 + (1 - \theta)v_2$</td>
</tr>
<tr>
<td>$\Pi^*$</td>
<td>$\alpha(\theta v_1 + (1 - \theta)v_2) + S - T - W$</td>
<td>$\theta v_1 + (1 - \theta)[xv_2 + S - T] - W$</td>
<td>$\theta v_1 + (1 - \theta)v_2 - W$</td>
</tr>
</tbody>
</table>

Table 1

Summary of optimal return scenarios
associated with a return by a type (i) customer is \( S - (1 - z)v_i \) (because if a type (i) customer makes a return, then that customer receives a refund \( R \), but forfeits \( (1 - z)v_i \), that customer’s residual value of the product; and the retailer receives a salvage value \( S \), but forfeits \( R \)). Thus, from the social planner’s perspective, the retailer and type (i) customers should coordinate the processing of returns only if \( T < (1 - z)v_i \), which is the result indicated by Theorem 1. Note that although, in principle, the AR case can arise as the optimal return scenario, for it to occur, the parameters must be such that \( S > (1 - z)v_1 + T \). In other words, a used product would have to be worth more on the salvage market than it is worth to the retailer’s most desirable customer segment. Products that meet this strict condition likely would be candidates for lease agreements (rather than purchase agreements) or would be highly desirable to recyclers or remanufacturers.

Another implication of Theorem 1 is that \( R^* \) is not unique: Since the profit associated with a given return scenario does not change with \( R \), any value of \( R \) in the interval defining the optimal return scenario serves as \( R^* \). One possible reason for this phenomenon is that rational customers are not concerned with the refund price in isolation. Instead, they simultaneously consider both the product’s refund price and the product’s selling price to arrive at a purchase decision. But, from Lemma 1, the selling price is an increasing function of the refund price for the AR and SR cases (which correspond to the two cases in which customers actually make returns). Thus, any decrease in the refund price is offset by a corresponding decrease in the selling price. Therefore, any number of (refund price, selling price) combinations will elicit the same customer behavior from the market, and thereby provide the same expected profit to the retailer. Correspondingly, there exists an infinite number of optimal refund alternatives. Note, however, when the model is generalized to a continuum of customers in Section 5, we find that a unique \( R^* \) exists in the SR scenario.

Further insight comes from comparing the optimal refund policy to customers’ costs. We state these comparisons formally as Theorems 2 and 3, and discuss the results in turn.

**Theorem 2.** If AR is the optimal return scenario, then
(i) \( R^* < p^* \) and
(ii) \( p^* + c - ax_1 < R^* < p^* + c - ax_2 \).

Part (i) of Theorem 2 is intuitive since it corresponds to a situation in which all customers return a purchased product. In other words, if the AR case is optimal, then the retailer really is not a merchant at all; instead, the retailer is a lessor. In such a scenario, \( p^* - R^* \) represents the retailer’s optimal per unit rental price, which must be greater than the retailer’s net per unit operating cost \( (W - S > 0) \) for the retailer to be in business at all.

Part (ii) of Theorem 2 can be interpreted to mean that only matched customers receive a “good deal” when the retailer is in the rental business. To see this, notice that a matched customer’s total cost of renting the product \( (p^* - R^* + c) \) is strictly less than the value that the customer receives from renting the product \( (zv_1) \). However, the surplus enjoyed by matched customers comes at the expense of mismatched customers, who end up with out-of-pocket expenses that exceed the value received from renting the product. Again, this is rather intuitive since, recall, matched customers receive positive utility from their dealings with the retailer while mismatched customers receive negative utility, with the balancing factor being the probability of match.

**Theorem 3.** If either SR or NR is the optimal return scenario, then
(i) it is possible to have \( R^* > p^* \) but
(ii) \( R^* < p^* + c - ax_2 \).

Part (i) of Theorem 3 indicates that the refund price can exceed the purchase price in an optimal scenario. This is not very surprising for the NR case since, in that scenario, the refund price is merely an empty
promise on the part of the retailer: No customers will be returning the product anyway (because, for example, \( c \) is prohibitively large). However, it is appealing to find that a refund price in excess of the purchase price can be optimal for the SR case, which corresponds to the scenario in which only mismatched customers return. In other words, it is possible to have an optimal scenario in which matched customers benefit more from keeping a product than from renting it, yet mismatched customers are reimbursed the full purchase price (and then some) after learning that the product does not fit their needs to the extent expected. A refund price that exceeds a product’s purchase price is sometimes seen in practice. Practical examples of companies that exploit such an opportunity include Dexter Shoes, which not only refunds the purchase price of a returned product, but also pays for shipping when the product is purchased from a catalog, and DiamondSafe.com, which reimburses the customer $10 more than the original purchase price.

Part (ii) of Theorem 3 parallels part (ii) of Theorem 2. Thus, although it is possible for mismatched customers to receive a full refund of their purchase price as well as partial subsidization of their costs incurred in completing a return, they never will be reimbursed fully for their trouble. However, it is interesting to note that it is possible, though not guaranteed, to have \( R^* > p^* + c - \alpha v_1 \) when either SR or NR is the optimal scenario. This is because although the retailer might provide a fair offer for making returns, it simply might be more worthwhile for matched customers to continue to extract utility from the product rather than to exercise their option of returning the product for a refund.

Theorems 2 and 3 show that, in many situations, it is not necessary to set the return price equal to the retail price, although this seems to be common in practice and in the literature (e.g., Davis et al., 1995, 1998). However, examples of setting the two prices separately exist as well, both in the literature (e.g., Hess et al., 1996) and in practice. One way that retailers set the two prices separately is to charge customers non-refundable shipping and handling costs at the time of purchase (still advertising a “full money-back guarantee”), which ultimately means that the return price is less than the original purchase price.

We close this section with a brief sensitivity analysis of the retailer’s optimal return policy to highlight additional insights. To that end, Theorem 4 summarizes effects on \( n^* \) and \( p^* \), and Theorem 5 summarizes effects on \( II^* \).

**Theorem 4.** The optimal return policy is such that

(i) if a change in \( T \) produces a change in \( n^* \), then it also produces a change in \( p^* \) — moreover, the changes in \( n^* \) and \( p^* \) are in the same direction;

(ii) if a change in \( \theta \) produces a change in \( n^* \), then it also produces a change in \( p^* \) — moreover, the changes in \( n^* \) and \( p^* \) are in the opposite direction;

(iii) if a change in either \( v_1 \) or \( v_2 \) produces a change in \( n^* \), then it also produces a change in \( p^* \) — moreover, the changes in \( n^* \) and \( p^* \) are in the same direction.

**Theorem 5.** \( II^* \) is non-increasing in \( T \), and is strictly increasing in \( \theta, v_1, \) and \( v_2 \).

One interesting implication of Theorem 4 is that matched customers share the cost of any effort made to improve the retailer’s overall returns system, even though any such improvement primarily benefits mismatched customers. In particular, when more efficient logistics (as represented by a decrease in \( T \) yields an increase in the expected number of returns, matched customers end up paying more (part (i)). Likewise, when more effective marketing (as represented by an increase in \( \theta \) yields a decrease in the expected number of returns, matched customers still end up paying more (part (ii)). As a result, the higher price resulting from an improvement to the retailer’s return system can be interpreted as an insurance premium from the perspective of matched customers: Since customers do not know ex ante whether or not the product will fit their needs, matched customers pay a premium to cover the risk that they might need the convenience of an
improved returns system. Interestingly, however, if a marketing effort is targeted toward increasing customers’ perceptions of the product’s value (as represented by an increase in $v_1$, $v_2$ or both), instead of toward better segmenting the market, then matched customers actually can experience a price discount if the marketing campaign ultimately yields a decrease in the expected number of returns (part (iii)).

Theorem 4 is useful for understanding the effects of perturbations that result in changes to the expected number of returns. However, effects that do not lead to changes in the expected number of returns are also interesting because they help draw parallels to results in the literature. In particular, it is interesting to consider the sensitivity of $p^*$, which represents the final price paid to the retailer by customers who do not make returns, and of $p^* - R^*$, which represents the final price paid to the retailer by customers who do make returns, relative to incremental changes in model parameters, assuming that the perturbations do not change the expected number of returns. To that end, we arbitrarily set $R^*$ equal to its lower limit for each return scenario (recall, $R^*$ is not unique), and summarize the sensitivity results in Table 2 (which follows directly from Table 1).

In general, Table 2 synthesizes and extends the cumulative results of related B2C product-returns models. In particular, Table 2 is consistent with the results of Davis et al. (1995), who find, for their model, that a money-back guarantee (i.e., $p^* = R^*$) is most applicable when $\theta$ is low. It also is consistent with the results of Hess et al. (1996), who find, for their model, that $p^* - R^*$ is increasing in $\theta$ and $v_1$, but is non-increasing in $c$; and that $p^*$ is increasing in $v_1$. Moreover, Table 2 is consistent with the results of Davis et al. (1998), who observed similar patterns for a case in which heterogeneous customer valuations are characterized by a uniform distribution. Thus, Table 2, which is derived from a simple and succinct model of the customer returns process, effectively extends similar results of previous researchers, while providing new insights.

Finally, we point out that the retailer will realize higher operational profits either if cost parameters (i.e., $T = c + h$) are decreased or if one or more market parameters (i.e., $\theta$, $v_1$, or $v_2$) are increased (Theorem 5). Thus, improvements in logistics, in marketing, or in both warrant obvious consideration.

### 4. Extended model: Optimal marketing and logistics strategy

In this section, we establish the jointly optimal investment scenario for logistics and marketing by extending the base model to include $\theta$ and $T$ as decision variables that can be set by the retailer through corresponding investments.

---

Table 2
Sensitivity analysis for perturbations that do not alter the optimal return scenario

<table>
<thead>
<tr>
<th></th>
<th>$c^*$</th>
<th>$\theta$</th>
<th>$v_1$</th>
<th>$v_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^*$</td>
<td>AR: 0</td>
<td>AR: +</td>
<td>AR: +</td>
<td>AR: +</td>
</tr>
<tr>
<td>SR: 0</td>
<td>SR: +</td>
<td>SR: +</td>
<td>SR: +</td>
<td></td>
</tr>
<tr>
<td>NR: 0</td>
<td>NR: +</td>
<td>NR: +</td>
<td>NR: +</td>
<td></td>
</tr>
<tr>
<td>$p^* - R^*$</td>
<td>AR: -</td>
<td>AR: +</td>
<td>AR: +</td>
<td>AR: +</td>
</tr>
<tr>
<td>SR: -</td>
<td>SR: +</td>
<td>SR: +</td>
<td>SR: ±</td>
<td></td>
</tr>
<tr>
<td>NR: 0</td>
<td>NR: +</td>
<td>NR: +</td>
<td>NR: +</td>
<td></td>
</tr>
</tbody>
</table>

(+•) denotes a positive correlation; (−•) denotes a negative correlation; (0•) denotes no correlation.

Bold cells indicate observations that are consistent with the literature; non-bold cells indicate new observations.

• Both $p^*$ and $R^*$ depend on $T$ only through $c$; each is independent of $h$.

• If $\alpha > 0$, then the relationship is (+•); if $\alpha < 0$, then the relationship is (−•).

---

3 Details of the derivation of Table 2 are available from the authors upon request.
Toward that end, we assume that $\theta$ is a factor that can be influenced by the retailer through a marketing initiative designed to increase the average number of matches per sale. One way to interpret this assumption is to consider a retailer who operates in a business environment in which the total pool of potential customers in the market is large relative to the retailer’s capacity. In such a scenario, the retailer’s total sales is a constant that is equal to her capacity. Since the retailer sells her entire stock (which, without loss of generality, we assume to be equal to one), but the product she sells does not meet the needs of the entire pool of potential customers, there exists the opportunity for the retailer to improve her signaling effort regarding the product’s properties and features so that the market becomes better segmented. As a result, some of the customers can recognize a priori that the product does not fit their needs, and thus exit the market, thereby increasing the density of the matched population that remains in the pool.\(^4\) (Note that, in the extreme case of perfect information, the market would become perfectly segmented so that only the matched population remained.) To capture this effect, we define $K(\theta)$ as the investment required in the marketing campaign so that the probability that the product will match a given customer’s pre-purchase expectations increases from $\theta_0$ to $\theta$ ($\theta_0$ denotes the equilibrium value of $\theta$ that would result if no marketing investment were made). We assume that $K(\theta_0) = 0$ and that $K(\theta)$ is continuously differentiable, increasing, and convex for $\theta \in [0, 1]$.

Similarly, we assume that $T$ is a factor that can be influenced by the retailer through an investment in a logistics process designed to accomplish either or both of two effects: simplified return procedures for the customer (which effectively reduces $c$) or more efficient processing procedures for the retailer (which effectively reduces $h$). To capture this effect, we define $J(T)$ as the investment required to improve the retailer’s logistics process so that the total cost of shipping and handling decreases from $T_0$ to $T$ ($T_0$ denotes the equilibrium value of $T$ that would result if no logistics investment were made). We assume that $J(T_0) = 0$ and that $J(T)$ is continuously differentiable, decreasing, and convex for $T \in [0, T_0]$.

Given the specification of these investment functions, and given that $p$ and $R$ are chosen optimally for any given values of $\theta$ and $T$ (in accordance with Theorem 1), the retailer’s joint optimization problem is to choose $\theta_0 \leq \theta \leq 1$ and $0 \leq T \leq T_0$ to maximize where, from Table 1,

$$
\Pi^*(\theta, T) = \begin{cases}
\begin{align*}
&x(\theta v_1 + (1 - \theta)v_2) + S - T - W \\
&\text{for } T < S - (1 - z)v_1, \quad \text{(AR)},
\end{align*}
\end{cases}
\begin{cases}
\begin{align*}
&\theta v_1 + (1 - \theta)(S - S - T) - W \\
&\text{for } S - (1 - z)v_1 \leq T < S - (1 - z)v_2, \quad \text{(SR)},
\end{align*}
\end{cases}
\begin{cases}
\begin{align*}
&\theta v_1 + (1 - \theta)v_2 - W \\
&\text{for } S - (1 - z)v_2 \leq T. \quad \text{(NR)}.\end{align*}
\end{cases}
$$

Notice that for any given value of $T$, $\Pi^*(\theta, T)$ is differentiable everywhere as a function of $\theta$; but, for a given value of $\theta$, $\Pi^*(\theta, T)$ is not differentiable everywhere as a function of $T$. Consequently, it is convenient to approach the retailer’s joint optimization problem in two steps: First, three candidate solutions can be obtained by establishing what the retailer’s best policy would be if it were constrained to operate in each of

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\(^4\)To illustrate this notion of “better segmentation”, consider a product with two potential attributes (a, b), and a market consisting of four possible segments based on whether they prefer an attribute (1) or not (0): (1, 0), (0, 1), (1, 0), (1, 1). Our retailer has a specific version of the product, say (1, 0). Notice that this means there is a total of one matched segment and three mismatched segments. Thus, without effective information flow to the customers, the probability of match for any sale is 1/4. However, let us say that the retailer (after incurring some cost), successfully communicates to the market that her product does carry attribute “a”. Then, customer types (0, 1) and (0, 0) realize that they are mismatched and leave the market, thereby increasing the probability of match for a given sale to 1/2. Note that the result of this marketing effort does not change the total number of matched segments (the number of matched segments is still one). Instead, it reduces the overall size of the market (from four possible customer segments to two) since some mismatched customers leave the market.
three possible regions (AR, SR, NR). Then, the retailer’s optimal policy can be determined by comparing the expected profit associated with each of the three candidate solutions.

Accordingly, let \( \theta^* \) and \( T^* \) denote the retailer’s jointly optimal choice for \( \theta \) and \( T \), respectively. Moreover, for \( i = AR, SR, \) and \( NR \), let \( \theta_i \) and \( T_i \) denote the retailer’s jointly optimal choice for \( \theta \) and \( T \), respectively, given that the retailer restricts its decision space to region \( i \). Then, for \( i = AR, SR, \) and \( NR \), \((\theta_i, T_i)\) is the solution to the following optimization problem (where \( \tau(i) \) denotes the range of \( T \’s \) that are feasible for scenario \( i \) to be optimal):

\[
\max_{\theta \leq \theta \leq 1 \quad 0 \leq T \leq T_0 \quad T \in \tau(i)} \pi_i(\theta, T) = \Pi_i(\theta, T) - J(T) - K(\theta).
\]

Correspondingly, \((\theta^*, T^*) = \arg\max\{\pi_{AR}(\theta_{AR}, T_{AR}), \pi_{SR}(\theta_{SR}, T_{SR}), \pi_{NR}(\theta_{NR}, T_{NR})\}\). Note that if the intersection between \( 0 \leq T \leq T_0 \) and \( T \in \tau(i) \) is empty for region \( i \), then region \( i \) produces no candidate solution; hence, in such a case, the region can be ignored from consideration for determining \((\theta^*, T^*)\).

The following theorem characterizes \((\theta_i, T_i)\) for \( i = AR, SR, \) and \( NR \). The theorem basically establishes three things: First, determining \((\theta_{AR}, T_{AR})\) and \((\theta_{SR}, T_{SR})\) is relatively simple because necessary and sufficient conditions follow directly from the general assumptions that \( J(T) \) is decreasing and convex and that \( K(\theta) \) is increasing and convex. Second, although determining \((\theta_{SR}, T_{SR})\) is less straightforward because \( \pi_{SR}(\theta, T) \) is not necessarily jointly concave, the best choice for \( \theta \) in region \( SR \) can be determined uniquely for a given \( T \). Thus, the optimization problem associated with region \( SR \) can be reduced to a single-variable search for \( T_{SR} \). And third, although, as a worst case scenario, finding \( T_{SR} \) would require an exhaustive search over \( T \’s \) feasible domain, a fairly general sufficiency condition is provided to ensure the unique characterization of \( T_{SR} \).

**Theorem 6.** For \( i = AR, SR, \) and \( NR \), the retailer’s jointly optimal choice for \( \theta \) and \( T_i \) can be characterized as follows:

(i) \( \theta_{AR} = \max\{\theta_0, \min\{\theta_{AR}, 1\}\} \), where \( \theta_{AR} \) is the unique value of \( \theta \) that satisfies \( K'(\theta) = d(v_1 - v_2) \); and \( T_{AR} = \max\{0, \min\{T_{AR}, T_0\}\} \), where \( T_{AR} \) is the unique value of \( T \) that satisfies \( J'(T) = -1 \).

(ii) \( \theta_{NR} = \max\{\theta_0, \min\{\theta_{NR}, 1\}\} \), where \( \theta_{NR} \) is the unique value of \( \theta \) that satisfies \( K'(\theta) = (v_1 - v_2) \); and \( T_{SR} = T_0 \).

(iii) \( \theta_{SR} = \max\{\theta_0, \min\{\theta_{SR}, 1\}\} \), where \( \theta_{SR} \) is the unique value of \( \theta \) that satisfies \( K'(\theta) = v_1 - (v_2 - S + T) \); and \( T_{SR} = \max\{0, \min\{T_{SR}, T_0\}\} \), where \( T_{SR} = \arg\max\{\pi_{SR}(\theta_{SR}(T), T)\}\).

(iv) If \( K''(\theta)J''(T) > 1 \) for all values of \( \theta \) and \( T \) that jointly satisfy \( K'(\theta) = v_1 - (v_2 - S + T) \) and \( J'(T) = -1(1 - \theta) \), then \( \pi_{SR}(\theta_{SR}(T), T) \) is pseudo-concave in \( T \); hence \( T_{SR} \) can be determined as the unique value of \( T \) that satisfies \( J'(T) = -1(1 - \theta_{SR}(T)) \).

5. Discrete vs. continuous “matchability”

In this section, we generalize the characterization of the match between the retailer’s product and any given customer’s pre-purchase expectations to allow the “level of match” to fall along a continuum. In other words, we relax the assumption that a customer’s ex post valuation of the product be either \( v_2 \) or \( v_1 \), and instead assume that the valuation, \( V \), can fall anywhere within the interval \([v_2, v_1]\). The actual value of \( V \) depends on \( \theta \), which we now can interpret more broadly to represent how well the product’s properties and features actually match the customer’s needs. Accordingly, to generalize the model analyzed earlier, let \( V \) be characterized over the interval \([v_2, v_1]\) by the distribution function \( F(v|\theta) \). We assume that
$\partial F(v|\theta) / \partial \theta \leq 0$ to signify that a higher $\theta$ corresponds to a higher $V$, in expectation. Note that if $F(v|\theta)$ is a Bernoulli distribution so that $V = v_1$ with probability $\theta$ and $V = v_2$ with probability $1 - \theta$, then the general model reduces to the special case solved and analyzed in Sections 3 and 4. Note also that if $F(v|\theta) = v$ so that $V$ is uniformly distributed over $[0, 1]$, independent of $\theta$, then the general model reduces to the special case solved and analyzed in Davis et al. (1998).

Given that $V \sim F(v|\theta)$, the customer’s expected utility function, (1), generalizes to

$$U(p, R) = \int_{v_1}^{v_2} \max\{v, xv + R - c\} f(v|\theta)\, dv - p = E[V|\theta] + (1 - z) \int_{v_2}^{\infty} (z - v)f(v|\theta)\, dv - p,$$

(5)

where $E[V|\theta]$ denotes the expected value of $V$, and $z \equiv (R - c)/(1 - x)$. Note that $z$ can be interpreted as the minimum valuation of the product required for the customer to decide not to return the product. That is, the customer’s return decision is to return the product if and only if the realized value of $V$ is less than $z$. Accordingly, $F(z|\theta)$ represents the probability that a customer will return the product.

Note also that $z$ serves as a proxy for $R$; thus, the retailer’s decision problem can be written in terms of choosing the $p$ and the $z$ (instead of the $p$ and the $R$) that maximizes its expected profit. Hence, given (5), the retailer’s decision problem, originally specified as (2) and (3), can be written more generally as follows:

$$\max_{z, p} \quad p - W - ((1 - x)z + T - S)F(z)$$

s.t. $\quad p \leq E[V|\theta] + (1 - z) \int_{v_2}^{\infty} (z - v)f(v|\theta)\, dv.$

(6)

The solution to (6) is summarized by Table 3, which directly extends the solution to the special case (Table 1) with one exception: In general, if SR is the optimal return scenario, then $R^*$ is unique, which is intuitive. It is interesting to note, however, that if SR is the optimal return scenario, then $R^* = S - h$, which indicates that, in general, it is in the retailer’s best interest to pass all of its net cash flow associated with a return ($S - h$) on to the customer.

Given the parallel between Tables 3 and 1, it is not surprising that Theorems 2–5 also continue to hold (the proofs are available from the authors on request) for the general specification of $F(v|\theta)$. Moreover, the procedure for determining $\theta^*$ and $T^*$ for a general $F(v|\theta)$, although more difficult to implement from a computational standpoint, also parallels the corresponding procedure for the special case in which $F(v|\theta)$ is a Bernoulli distribution. In particular, $\theta^*$ and $T^*$ can be determined for a general $F(v|\theta)$ by first deriving the three candidate solutions ($\theta_{AR}, T_{AR}$), ($\theta_{SR}, T_{SR}$), and ($\theta_{NR}, T_{NR}$), and then making a direct comparison of the expected profits associated with each of the three candidate solutions. Like in the special case, both ($\theta_{AR}, T_{AR}$) and ($\theta_{NR}, T_{NR}$) can be derived directly from analysis of first-order conditions and the constraints $\theta \geq \theta_0$ and $T \leq T_0$. Likewise, $\theta_{SR}$ can be determined efficiently for any given $T$. However, whether or not a more exhaustive search is required to determine $T_{SR}$ depends, in general, not only on the specifications of $J(T)$ and $K(\theta)$, but also on the specification of $F(v|\theta)$.

Table 3
Summary of optimal return scenarios for general case

<table>
<thead>
<tr>
<th>Return scenario</th>
<th>AR</th>
<th>SR</th>
<th>NR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Optimality condition</strong></td>
<td>$T &lt; S(1 - z)c_1$</td>
<td>$S(1 - z)c_1 \leq T &lt; S(1 - z)c_2$</td>
<td>$S(1 - z)c_2 \leq T$</td>
</tr>
<tr>
<td>$R^*$</td>
<td>any $R$ s.t. $R &gt; (1 - z)c_1 + c$</td>
<td>$R = S - h$</td>
<td>any $R$ s.t. $R \leq (1 - z)c_2 + c$</td>
</tr>
<tr>
<td>$n^*$</td>
<td>$1$</td>
<td>$F([S - T]/(1 - z)]$</td>
<td>$0$</td>
</tr>
<tr>
<td>$p^*$</td>
<td>$xE[V</td>
<td>\theta] + R - c$</td>
<td>$E[V</td>
</tr>
<tr>
<td>$\Pi^*$</td>
<td>$xE[V</td>
<td>\theta] + S - T - W$</td>
<td>$E[V</td>
</tr>
</tbody>
</table>
6. The cost of inefficient decision-making

The coordination model of Section 4 assumed that a rational decision-making process is followed in the organization. What if this were not the case? For example what if one department made its investment decision without considering or anticipating the investment decision of the other department? Obviously, such uncoordinated behavior would result in sub-optimal decisions. Nonetheless, evidence from literatures in organization theory indicates that such uncoordinated behavior does exist in practice.

The effects of organizational politics on decision-making has been prevalent in organization theory literature for some time (March and Simon, 1958; Cyert and March, 1963). This political perspective views an organization as a loose structure of interests and demands, competing for organizational attention and resources, and resulting in conflicts that are never completely resolved. Who gets to make important decisions could be based upon the relative power of individuals and departments. As Dean and Sharfman (1996) note, “Also, a course of action that is promising in light of the environment may be eliminated because of the opposition of a powerful individual. Thus, political processes may rule out viable choices, further reducing the likely success of the strategic decisions they produce”.

Studies by Pfeffer (1981), Eisenhardt and Bourgeois (1988), and Dean and Sharfman (1996) have all suggested or reported a link between politics and unsuccessful decisions. Eisenhardt and Zbaracki (1992) present a review of 14 published papers of empirical research that further support these findings. Therefore, political play is not only prevalent in the decision-making process, but it is also a concern for rational or optimal decision-making, reducing the effectiveness of the decision-making process, and in turn affecting the success of the organization.

In this section, we explore the cost of such inefficiency by considering one possible characterization of uncoordinated decision-making by logistics and marketing. To that end, we compare the results of Section 4 to two alternative scenarios. Each alternative scenario is defined as an organizational reality in which either logistics or marketing, as the more powerful department (from a political perspective), makes an investment decision without considering or anticipating any potential investment action on the part of the less powerful department. We find that, for each alternative scenario, the more politically-powerful department achieves an inflated investment (as compared to the corresponding optimal investment) of the less powerful department. We find that, for each alternative scenario, the more politically-powerful department settles for a reduced investment.

To focus the analysis of this section, we place two restrictions on the input parameters of our model. First, we make the natural assumption that \( S \leq (1 - \alpha)v_1 \), which, recall, means that a used product is worth more to the retailer’s most desirable customers than it is worth on the salvage market. Since this condition assures that the AR case cannot arise as the optimal return scenario, it allows us to focus on questions of system improvement rather than on the special-case question of whether or not a retailer should exit the merchandising business and enter the leasing business. Second, we make the assumption that \( T_0 < S - (1 - \alpha)v_2 \), which means that the retailer’s optimal return scenario if no investment is made in the logistics system is SR. Since \( T \) is defined only for \( T \leq T_0 \), this condition assures that the NR case cannot arise as the optimal return scenario; hence, it allows us again to focus on questions of system improvement rather than on the special-case question of whether or not a retailer should invest in establishing a product returns system (and thereby move from NR to SR).

Given that \( S - (1 - \alpha)v_1 < 0 \leq T \leq T_0 < S - (1 - \alpha)v_2 \), the retailer’s feasible decision space is restricted to the SR region. Thus, if the retailer coordinates its investment decisions, its optimal strategy is \( \theta^* = \theta_{SR} \) and \( T^* = T_{SR} \). From parts (iii) and (iv) of Theorem 6, \( \theta^* \) and \( T^* \) simultaneously satisfy \( \theta^* = \max\{\theta_0, \min\{\theta(T^*), 1\}\} \) and \( T^* = \max\{0, \min\{T(\theta^*), T_0\}\} \), where \( \theta(T) \) is defined as any value of \( \theta \) that satisfies

\[
K'(\theta) = v_1 - \alpha v_2 - S + T
\]
and $T(\theta)$ is defined as any value of $T$ that satisfies

$$J'(T) = -(1 - \theta).$$

(8)

Notice from (7) that since $K'(\theta)$ is an increasing function by assumption, $\theta(T)$ increases as $T$ increases. Similarly, since $J'(T)$ is an increasing function by assumption, (8) implies that $T(\theta)$ increases as $\theta$ increases.

If the retailer does not coordinate its investment decisions, then the resulting (sub-optimal) choices for $\theta$ and $T$ depend on the political landscape within the retailer:

**Logistics dominates.** This scenario is defined as the alternative in which logistics is the more powerful department. As such, logistics chooses $T$ without regard for how that choice might affect the marketing department. Specifically, this inefficient decision scenario is characterized by a logistics department that treats $\theta$ as a constant (equal to $\theta_0$) when determining $T$. However, after $T$ is established, marketing chooses $\theta$, given logistics’ choice for $T$. We define the $T$ and $\theta$ obtained under this inefficient decision scenario as $T_L$ and $\theta_L$, respectively. Accordingly, $T_L = \max\{0, \min\{T(\theta_0), T_0\}\}$, where $T(\theta_0)$ is defined implicitly by (8), and $\theta_L = \max\{\theta_0, \min\{\theta(T_L), 1\}\}$, where $\theta(T_L)$ is defined implicitly by (7). Notice that $T_L = \max\{0, \min\{T(0^*), T_0\}\} = T^*$ since $T(0)$ is increasing in $\theta$ and $\theta_0 \leq 0^*$. Consequently, $\theta_L = \max\{0, \min\{\theta(T^*), 1\}\} = 0^*$ since $\theta(T)$ is increasing in $T$ and $T_L \leq T^*$.

**Marketing dominates.** This scenario is defined as the complementary alternative in which marketing is the more powerful department. As such, marketing chooses $\theta$ without regard for how that choice might affect the logistics department. Specifically, this inefficient decision scenario is characterized by a marketing department that treats $T$ as a constant (equal to $T_0$) when determining $\theta$. However, once $\theta$ is established, logistics chooses $T$, given marketing’s choice for $\theta$. We define the $T$ and $\theta$ obtained under this strategy as $T_M$ and $\theta_M$, respectively. Accordingly, $\theta_M = \max\{\theta_0, \min\{\theta(T_0), 1\}\}$, where $\theta(T_0)$ is defined implicitly by (7), and $T_M = \max\{0, \min\{T(\theta_M), T_0\}\}$, where $T(\theta_M)$ is defined implicitly by (8). Notice that $\theta_M \geq \max\{0, \min\{T(0^*), 1\}\} = 0^*$ since $\theta(T)$ is increasing in $T$ and $T_0 \geq T^*$. Consequently, $T_M \geq \max\{0, \min\{T(0^*), T_0\}\} = T^*$ since $T(\theta)$ is increasing in $\theta$ and $\theta_M \geq 0^*$.

To summarize, analysis of these non-coordinated scenarios yields the following theorem:

**Theorem 7.** Let $(T^*, 0^*)$ denote the retailer’s optimal investment policy under coordination, $(T_L, \theta_L)$ denote the retailer’s non-coordinated outcome if logistics dominates, and $(T_M, \theta_M)$ denote the retailer’s non-coordinated outcome if marketing dominates. Then,

(i) $T_L \leq T^* \leq T_M$; hence, $J(T_L) \geq J(T^*) \geq J(T_M)$ and

(ii) $\theta_L \leq \theta^* \leq \theta_M$; hence $K(\theta_L) \leq K(\theta^*) \leq K(\theta_M)$.

The relationships between the investment functions follow because $J(T)$ is decreasing in $T$ and $K(\theta)$ is increasing in $\theta$, by assumption. Theorem 7 indicates, in general, that if the retailer does not coordinate its investment decisions, then it will tend to over-invest in the department that is more powerful (since $J(T_L) \geq J(T^*)$ and $K(\theta_M) \geq K(\theta^*)$), and it will tend to under-invest in the department that is less powerful (since $K(\theta_L) \leq K(\theta^*)$ and $J(T_M) \leq J(T^*)$).

7. Conclusions

In this paper we developed an integrated approach for analyzing logistics and marketing decisions within the context of designing an optimal returns system for a retailer servicing two distinct market segments. One contribution of the paper is the policy implications of an optimal refund policy. In particular, we show
that the optimal refund price is not unique and is such that if both market segments return a purchased product, then neither segment will receive a full money-back refund, and only matched customers will extract enough value from the product during the trial period to offset their total net cost of purchase and return. However, if one or both segments do not return a purchased product, then a refund premium over the purchase price is possible, but the refund premium will not be enough to offset a customer's total net cost of purchase and return. We also show that any improvement to the returns system that results in increased logistical efficiency or marketing effectiveness will be accompanied by an increase in the selling price of the product. In addition, we illustrate how our model can be generalized to the case in which a customer's ex post valuation of the product falls along a continuum.

A second contribution of the paper is the insights corresponding to situations in which the retailer fails to coordinate her logistics and marketing investment decisions. In particular, we show that if the retailer does not coordinate its logistics and marketing efforts to improve the overall returns system, then she will tend to over-invest in the more politically-powerful department and under-invest in the less politically-powerful department.

A third contribution of the paper is the presentation of a streamlined model of the customer-returns process that effectively synthesizes and extends key results of previous literature on money-back guarantees, which typically assumes that parameters are such that not everyone will return purchased products.

We have designated $R$, the product refund price, as the decision proxy to represent the retailer's refund policy, and have assumed that $z$, the ratio of the trial period to the useful lifetime of the product, is exogenous. However, $z$ easily can be incorporated as a decision variable as well. Everything else being equal, the retailer prefers a higher $z$ since its profit is non-decreasing as a function of $z$. Therefore, if no further adjustment to the model is made, then the optimal $z$ is 1, which is intuitive because $S$, the retailer's salvage value, is constant. More realistically, if $z$ is a decision variable, then $S$ should be a non-decreasing function of $z$ to capture the notion that the longer a product is left in the possession of an original purchaser, the less it is worth in salvage upon its return. Conceptually, however, making $S$ a function of $z$ does not complicate the analysis. For example, if $S(z) = (1 - z)s$, where $s$ is a constant, first the optimal $z$ can be found and then the analysis of this paper follows as presented.

We have used an increasing $\theta$ to represent a marketing improvement in the retailer's integrated product returns system. Alternatively, either or both of $v_1$ and $v_2$ can be designated as the marketing decision variable. Since the retailer's profit is non-decreasing both as a function of $v_1$ and as a function of $v_2$, we expect similar results to apply. We also have assumed that $T$, the total shipping and handling cost associated with product returns, is linear in the number of returns. One extension to the model developed here would be to incorporate a fixed cost or some other form of economies of scale in the logistics system.

Finally, the product returns system studied here can be more broadly interpreted. We interpreted a returned product to mean that a customer's needs are not fulfilled in some way. However, a returned product also could be interpreted as the logical result of different customers having different needs. For example, the design of a standard "lease with the option to buy" contract to offer to a market consisting of two segments, one comprising customers who will not exercise the buy option at the conclusion of the lease period and the other comprising customers who will exercise the buy option at the conclusion of the lease period, is equivalent to the problem considered here.

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Appendix A

Proof of Theorem 1. From (4), \( \Pi_{AR} > \Pi_{SR} \) if and only if \( z(\theta v_1 + (1 - \theta)v_2) + S - T - W > \theta v_1 + (1 - \theta)[v_2 + S - T] - W \). This is true if and only if \( S - (1 - \alpha)v_1 > T \). Similarly, \( \Pi_{SR} > \Pi_{NR} \) if and only if \( S - (1 - \alpha)v_1 > T \). Therefore, if \( T < S - (1 - \alpha)v_1 < S - (1 - \alpha)v_2 \), then \( \Pi_{AR} > \Pi_{SR} > \Pi_{NR} \). If \( S - (1 - \alpha)v_1 \leq T < S - (1 - \alpha)v_2 \), then \( \Pi_{SR} \geq \Pi_{AR} \) and \( \Pi_{SR} > \Pi_{NR} \). If \( S - (1 - \alpha)v_1 < S - (1 - \alpha)v_2 \leq T \), then \( \Pi_{NR} \geq \Pi_{SR} > \Pi_{AR} \). □

Proof of Theorem 2. Part (i). If AR is optimal, then \( \Pi_{AR} > 0 \). Thus, from Table 1, \( \Pi_{AR} = p^* - R^* + h + S - W > 0 \). Since \( W > S \) by assumption, this implies that \( p^* - R^* > h + (W - S) > 0 \).

Part (ii). From Table 1, \( p^* - R^* + c = z(\theta v_1 + (1 - \theta)v_2) \); thus \( x_2 < p^* - R^* + c < x_1 \).

Proof of Theorem 3. Part (i). We provide two examples to establish part (i). To that end, first consider the following example: \( v_1 = 1, v_2 = 0.4, \theta = 0.6, \alpha = 0.2, c = 0.2, h = 0.1, W = 0.75, \) and \( S = 0.65 \). Given these parameters, \( T = c + h = 0.3 \). Thus, \( -0.15 = S - (1 - \alpha)v_1 < T < S - (1 - \alpha)v_2 = 0.33, \) and \( \Pi_{SR} = \theta v_1 + (1 - \theta)[v_2 + S - T] - W = 0.022 > 0 \). Therefore, from Table 1, SR is the optimal return scenario. This implies that \( p^* = \theta v_1 + (1 - \theta)[v_2 + R - c] = 0.552 + 0.4R \), and that \( R^* \) is any \( R \) that satisfies \( 0.52 < (1 - \alpha)v_2 + c < R \leq (1 - \alpha)v_1 + c = 1.0 \). Thus, SR is optimal and \( R^* > p^* \) for any \( R \) that satisfies \( 0.92 < R \leq 1.12 \).

Next, consider the following example: \( v_1 = 1, v_2 = 0.4, \theta = 0.8, \alpha = 0.2, c = 0.8, h = 0.1, W = 0.6, \) and \( S = 0.4 \). Given these parameters, \( T = c + h = 0.8 \). Therefore, from Table 1, NR is the optimal return scenario. This implies that \( p^* = \theta v_1 + (1 - \theta)v_2 = 0.88, \) and that \( R^* \) is any \( R \) that satisfies \( R \leq (1 - \alpha)v_2 + c = 1.12 \). Thus, NR is optimal and \( R^* > p^* \) for any \( R \) that satisfies \( 0.88 < R \leq 1.12 \).

Part (ii). If SR is optimal, then, from Table 1, \( p^* = \theta v_1 + (1 - \theta)[v_2 + R^* - c], \) where \( R^* \leq (1 - \alpha)v_1 + c \). Therefore, \( p^* - x_2 - R^* + c = \theta v_1 - x_2 - R^* + c \geq \theta x_2 = 0 \).

If NR is optimal, then, from Table 1, \( p^* = \theta v_1 + (1 - \theta)v_2, \) and \( R^* \leq (1 - \alpha)v_2 + c \). Therefore, \( p^* - x_2 - R^* + c \geq \theta (v_1 - v_2) > 0 \).

Proof of Theorem 4. Let \( p_i \) and \( R_i \) represent \( p^* \) and \( R^* \), respectively, for the case in which \( i \) is the optimal return scenario (\( i = AR, SR, \) and \( NR \)). Then, from Table 1,

\[
R_{AR} > (1 - \alpha)v_1 + c \geq R_{SR} > (1 - \alpha)v_2 + c \geq R_{NR},
\]

\[
p_{AR} = z(\theta v_1 + (1 - \theta)v_2) + R_{AR} - c,
\]

\[
p_{SR} = \theta v_1 + (1 - \theta)[v_2 + R_{SR} - c],
\]

\[
p_{NR} = \theta v_1 + (1 - \theta)v_2.
\]

Thus,

\[
p_{AR} > z(\theta v_1 + (1 - \theta)v_2) + (1 - \alpha)[\theta v_1 + (1 - \theta)v_1]
\]

\[
> \theta v_1 + (1 - \theta)[v_2 + R_{SR} - c] = p_{SR} \quad (A.5)
\]

\[
> \theta v_1 + (1 - \theta)v_2 = p_{NR} \quad (A.6)
\]

Part (i). From Table 1, as \( T \) increases, the optimal return scenario moves from AR to SR to NR. Thus, an increase in \( T \) can cause \( n^* \) to change in only two ways: Either it can cause \( n^* \) to decrease from 1 to \( 1 - \theta \), which would be accompanied by a corresponding decrease in \( p^* \) from \( p_{AR} \) to \( p_{SR} \) (A.5); or it can cause \( n^* \) to
Proof of Theorem 5. As a function of $v$, $p^*$ approaches 0 when $v < 1$. Notice, both $p_{SR}$ and $p_{NR}$ are linearly increasing as functions of $v$. Thus, in this case, $p^*$ is continuous (piecewise linear) and increasing in $v$.

Part (ii). As $\theta$ increases, the optimal return scenario remains constant. Thus, an increase in $\theta$ can cause $n^*$ to change only if the optimal return scenario is $SR$. In such a case, $n^* = 1 - \theta$, which is decreasing in $\theta$; and $p^* = p_{SR}$, which is increasing in $\theta$, as shown below:

$$\frac{\partial p_{SR}}{\partial \theta} = v_1 - [xv_2 + R_{SR} - c] > v_1 - [xv_2 + (1 - \alpha)v_1] = \alpha(v_1 - v_2) > 0.$$  

Part (iii). First consider changes in $v_1$. There are two cases: Case 1. If $v_2 \geq (S - T)/(1 - \alpha)$, then $v_1 > v_2 > (S - T)/(1 - \alpha)$, which implies (from Table 1) that $NR$ is the optimal return scenario for all $v_1$. In this case, $n^*$ does not change as a result of changes in $v_1$. Case 2. If $v_2 < (S - T)/(1 - \alpha)$, then Table 1 implies that $AR$ is the optimal return scenario for $v_2 < v_1 < (S - T)/(1 - \alpha)$, and $SR$ is the optimal return scenario for $v_2 < (S - T)/(1 - \alpha) \leq v_1$. Thus, an increase in $v_1$ can cause $n^*$ to change only by causing it to decrease from 1 to $1 - \theta$. Such a drop would be accompanied by a corresponding decrease in $p^*$ from $p_{AR}$ to $p_{SR}$ (A.5).

Next consider changes in $v_2$. Again there are two cases: Case 1. If $v_1 \leq (S - T)/(1 - \alpha)$, then $v_2 < v_1 \leq (S - T)/(1 - \alpha)$, which implies (from Table 1) that $AR$ is the optimal return scenario for all $v_2$. In this case, $n^*$ does not change as a result of changes in $v_2$. Case 2. If $v_1 > (S - T)/(1 - \alpha)$, then Table 1 implies that $SR$ is the optimal return scenario for $v_2 < v_1 < (S - T)/(1 - \alpha)$, and $NR$ is the optimal return scenario for $(S - T)/(1 - \alpha) \leq v_2 < v_1$. Thus, an increase in $v_2$ can cause $n^*$ to change only by causing it to decrease from $1 - \theta$ to 0. Such a drop would be accompanied by a corresponding decrease in $p^*$ from $p_{SR}$ to $p_{NR}$ (A.6).

**Proof of Theorem 5.** From Table 1,

$$\Pi^* = \begin{cases} 
\Pi_{AR} = \alpha[v_1 + (1 - \theta)v_2] + S - T - W & \text{for } T < S - (1 - \alpha)v_1, \\
\Pi_{SR} = \theta v_2 + (1 - \theta)[xv_2 + S - T] - W & \text{for } S - (1 - \alpha)v_1 \leq T < S - (1 - \alpha)v_2, \\
\Pi_{NR} = \theta v_2 + (1 - \theta)v_2 - W & \text{for } S - (1 - \alpha)v_2 \leq T.
\end{cases}$$

As a function of $T$, both $\Pi_{AR}$ and $\Pi_{SR}$ are linear, and $\Pi_{NR}$ is constant. Moreover, the limit of $\Pi_{AR}$ as $T$ approaches $S - (1 - \alpha)v_1$ is equal to $\Pi_{SR}$, and the limit of $\Pi_{SR}$ as $T$ approaches $S - (1 - \alpha)v_2$ is equal to $\Pi_{NR}$. Thus, $\Pi^*$ is continuous (piecewise linear) and non-increasing in $T$.

To determine how $\Pi^*$ behaves as a function of $\theta$, there are three cases: Case 1. If $T < S - (1 - \alpha)v_1$, then $\Pi^* = \Pi_{AR}$ for all $\theta$. In this case, $\partial \Pi_{AR}/\partial \theta = \alpha(v_1 - v_2) > 0$. Case 2. If $S - (1 - \alpha)v_1 \leq T < S - (1 - \alpha)v_1$, then $\Pi^* = \Pi_{SR}$ for all $\theta$. In this case, $\partial \Pi_{SR}/\partial \theta = v_1 - [xv_2 + S - T] \geq v_1 - [xv_2 + (1 - \alpha)v_1] = \alpha(v_1 - v_2) > 0$. Case 3. If $S - (1 - \alpha)v_2 \leq T$, then $\Pi^* = \Pi_{NR}$ for all $\theta$. In this case, $\partial \Pi_{NR}/\partial \theta = v_1 - v_2 > 0$.

To determine how $\Pi^*$ behaves as a function of $v_1$, there are two cases: Case 1. If $v_2 \geq (S - T)/(1 - \alpha)$, then $\Pi^* = \Pi_{NR}$ for all $v_1$ (because $v_1 \geq v_2 \geq (S - T)/(1 - \alpha)$). In this case, $\partial \Pi_{NR}/\partial v_1 = \theta > 0$. Case 2. If $v_2 < (S - T)/(1 - \alpha)$, then

$$\Pi^* = \begin{cases} 
\Pi_{AR} & \text{for } v_2 < v_1 < (S - T)/(1 - \alpha), \\
\Pi_{SR} & \text{for } v_2 < (S - T)/(1 - \alpha) \leq v_1.
\end{cases}$$

Notice, both $\Pi_{AR}$ and $\Pi_{SR}$ are linearly increasing as functions of $v_1$. Moreover, the limit of $\Pi_{AR}$ as $v_1$ approaches $(S - T)/(1 - \alpha)$ is equal to $\Pi_{SR}$. Thus, in this case, $\Pi^*$ is continuous (piecewise linear) and increasing in $v_1$. 


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To determine how $\Pi^*$ behaves as a function of $v_2$, again there are two cases: Case 1. If $v_1 < (S - T)/(1 - x)$, then $\Pi^* = \Pi_{AR}$ for all $v_1$ (because $v_2 < v_1 < (S - T)/(1 - x)$). In this case, $\partial \Pi_{AR} / \partial v_2 = x(1 - 0) > 0$. Case 2. If $v_2 \geq (S - T)/(1 - x)$, then

$$
\Pi^* = \begin{cases} 
\Pi_{SR} & \text{for } v_2 < (S - T)/(1 - x) \leq v_1, \\
\Pi_{NR} & \text{for } (S - T)/(1 - x) \leq v_2 < v_1.
\end{cases}
$$

Notice, both $\Pi_{SR}$ and $\Pi_{NR}$ are linearly increasing as functions of $v_2$. Moreover, the limit of $\Pi_{SR}$ as $v_2$ approaches $(S - T)/(1 - x)$ is equal to $\Pi_{NR}$. Thus, in this case, $\Pi^*$ is continuous (piecewise linear) and increasing in $v_2$. □

**Proof of Theorem 6.** Part (i). If $i = AR$, then $\pi(\theta, T) = a(\theta v_1 + (1 - \theta)v_2) + S - T - W - J(T) - K(\theta)$. Thus, the first-order conditions for $\theta_{AR}$ and $T_{AR}$ are as follows:

$$
\frac{\partial \pi(\theta, T)}{\partial \theta} = a(v_1 - v_2) - K'(\theta) = 0 \quad \text{and} \quad \frac{\partial \pi(\theta, T)}{\partial T} = -1 - J'(T) = 0.
$$

Since both $K'(\theta)$ and $J'(T)$ are increasing functions by assumption, these first-order conditions are also sufficient for interior point solutions. Correspondingly, let $\theta_{AR}$ be the unique value of $\theta$ that satisfies $K'(\theta) = a(v_1 - v_2)$, and let $T_{AR}$ be the unique value of $T$ that satisfies $J'(T) = -1$. Then, $\theta_{AR}$ denotes the interior point solution for $\theta_{AR}$, if one exists; and $T_{AR}$ denotes the interior point solution for $T_{AR}$, if one exists. Therefore, taking the constraints $\theta_0 \leq \theta \leq 1$ and $0 \leq T \leq T_0$ into consideration, $\theta_{AR} = \max\{\theta_0, \min\{\theta_{AR}, 1\}\}$ and $T_{AR} = \max\{0, \min\{T_{AR}, T_0\}\}$.

Part (ii). If $i = NR$, then $\pi(\theta, T) = \theta v_1 + (1 - \theta)v_2 - W - J(T) - K(\theta)$. Thus,

$$
\frac{\partial \pi(\theta, T)}{\partial \theta} = v_1 - v_2 - K'(\theta) \quad \text{and} \quad \frac{\partial \pi(\theta, T)}{\partial T} = -J'(T) > 0.
$$

Analogous to part (i), since $K'(\theta)$ is increasing, $\partial \pi(\theta, T) / \partial \theta = 0$ is both necessary and sufficient for determining $\theta_{NR}$ if it is an interior-point solution. Therefore, $\theta_{NR} = \max\{\theta_0, \min\{\theta_{NR}, 1\}\}$, where $\theta_{NR}$ is the unique value of $\theta$ that satisfies $K'(\theta) = v_1 - v_2$. However, since $\partial \pi(\theta, T) / \partial T > 0$ for all $T$, the constraint $T \leq T_0$ implies that $T_{NR} = T_0$.

Part (iii). If $i = SR$, then $\pi(\theta, T) = \theta v_1 + (1 - \theta) [a v_2 + S - T] - W - J(T) - K(\theta)$, which is concave in $\theta$. Thus, for any given value of $T$, $\partial \pi(\theta, T) / \partial \theta = 0$ can be solved uniquely for $\theta$ as a function $T$. Accordingly, let $\theta(T)$ denote the solution to $\partial \pi(\theta, T) / \partial \theta = 0$. Then, $\theta(T)$ is the unique value of $\theta$ that satisfies $K'(\theta) = v_1 - a v_2 - S + T$. Next, let $\theta_{SR}(T)$ denote the optimal choice of $\theta$ if $T$ is fixed and $i = SR$. Then, given $T$ and taking the constraint $\theta_0 \leq \theta \leq 1$ into consideration, $\theta_{SR}(T) = \max\{\theta_0, \min\{\theta(T), 1\}\}$.

Next, substituting $\theta_{SR}(T)$ for $\theta$ in $\pi(\theta, T)$ reduces the retailer’s optimization problem for this region to a single-variable search over $T$:

$$
\max_{\theta \in [\theta_0, \theta_{SR}(T)]} \pi_{SR}(\theta_{SR}(T), T).
$$

Consequently,

$$
T_{SR} = \arg\max_{0 \leq T \leq T_0} \{\pi_{SR}(\theta_{SR}(T), T)\}.
$$

Hence,

$$
\theta_{SR} = \theta_{SR}(T_{SR}) = \max\{\theta_0, \min\{\theta(T_{SR}), 1\}\}.
$$

Part (iv). Recall from part (iii) that $\pi(\theta, T) = \theta v_1 + (1 - \theta) [a v_2 + S - T] - W - J(T) - K(\theta)$, $\theta_{SR}(T) = \max\{\theta_0, \min\{\theta(T), 1\}\}$, and $K'(\theta(T)) = v_1 - a v_2 - S + T$. Consequently, $d\theta(T) / dT = 1 / K''(\theta(T)) > 0$ and
\[
\frac{d\pi(\theta_{SR}(T), T)}{dT} = \frac{d\theta_{SR}(T)}{dT} \frac{\partial \pi(\theta, T)}{\partial \theta} \bigg|_{\theta = \theta_{SR}(T)} + \frac{\partial \pi(\theta_{SR}(T), T)}{\partial T}.
\]

However, for any given \( T \), the first term is zero because \( \frac{\partial \pi(\theta, T)}{\partial \theta} = 0 \) if \( \theta_{SR}(T) = \theta(T) \), by the definition of \( \theta(T) \); and \( \frac{d\theta_{SR}(T)}{dT} = 0 \) if \( \theta_{SR}(T) = \theta_0 \) or if \( \theta_{SR}(T) = 1 \). Thus,

\[
\frac{d\pi(\theta_{SR}(T), T)}{dT} = \frac{\partial \pi(\theta_{SR}(T), T)}{\partial T} = -(1 - \theta_{SR}(T)) - J'(T)
\]

and

\[
\frac{d^2\pi(\theta_{SR}(T), T)}{dT^2} = \frac{d\theta_{SR}(T)}{dT} - J''(T).
\]

Let \( \tilde{T} \) be any value of \( T \) that satisfies \( d\pi(\theta_{SR}(T), T)/dT = -(1 - \theta_{SR}(T)) - J'(T) = 0 \). Then, \( \tilde{T} \) is a possible interior point solution for \( T_{SR} \). There are three possibilities associated with \( \tilde{T} \): either \( \theta_{SR}(\tilde{T}) = \theta_0 \), \( \theta_{SR}(\tilde{T}) = \theta(\tilde{T}) \), or \( \theta_{SR}(\tilde{T}) = 1 \). If \( \tilde{T} \) is such that \( \theta_{SR}(\tilde{T}) = \theta_0 \), then \( d^2\pi(\theta_{SR}(T), T)/dT^2|_{T = \tilde{T}} = -J''(\tilde{T}) < 0 \), which implies that \( \tilde{T} \) would also satisfy the sufficiency condition for an interior point solution. Likewise, if \( \tilde{T} \) is such that \( \theta_{SR}(\tilde{T}) = 1 \), then \( d^2\pi(\theta_{SR}(T), T)/dT^2|_{T = \tilde{T}} = -J''(\tilde{T}) < 0 \), which again implies that \( \tilde{T} \) would also satisfy the sufficiency condition for an interior point solution. Finally, if \( \tilde{T} \) is such that \( \theta_{SR}(\tilde{T}) = \theta(\tilde{T}) \), then

\[
\frac{d^2\pi(\theta_{SR}(T), T)}{dT^2} \bigg|_{T = \tilde{T}} = \frac{1}{K''(\theta(\tilde{T}))} \left[ 1 - K''(\theta(\tilde{T})))J''(\tilde{T}) \right] < 0,
\]

where the inequality follows by the assumption that \( K''(\theta(\tilde{T})))J''(\tilde{T}) > 1 \). This implies that any \( \tilde{T} \) that is such that \( \theta_{SR}(\tilde{T}) = \theta(\tilde{T}) \) would also satisfy the sufficiency condition for an interior point solution. In other words, every possible value of \( T \) that satisfies the necessary condition for an interior point solution also satisfies the sufficiency condition. Consequently, there can be at most one such value of \( T \), namely \( T_{SR} \). In other words, \( \pi(\theta_{SR}(T), T) \) is pseudo-concave in \( T \), and \( T_{SR} \) is the unique value of \( T \) that satisfies \( J'(T) = -(1 - \theta_{SR}(T)) \). \( \square \)

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